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Research Article

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Marine protected areas for resilience and economic development

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Abstract – In this research, we attempt to give a comparative analysis of the space allocation of multiple-use marine protected areas (MPAs) including but not limited to the introduction of aquaculture in the area. Specifically, we consider the case where there is a need to develop MPAs for the conservation of the environment and ecological diversity. There is also a prevailing call for the establishment of aquaculture activities within the area to meet societal demands. Although aquaculture has negative externalities on MPAs, it helps to reduce the pressure on the capture fishery and increases the supply of fish. We develop a deterministic bioeconomic model that describes the transition dynamics and interrelationships of the systems. We find an optimal aquaculture size relative to the optimal size of MPAs that maximizes the overall economic and ecological benefits. Using numerical methods we determine the trajectory of optimal solutions, the recovery rate of the stocks in and outside the MPAs, and the expansion rate of the aquaculture. Sensitivity analysis was also performed to see the effect of a change in the parameters on the optimal solutions. The numerical results show that MPAs are resilient after the implementation of aquaculture. Moreover, the effectiveness of the optimized management system mainly depends on the cooperative planning between the capture fishery and aquaculture managers.

Keywords: Marine protected areas / aquaculture production / space allocation / growth rate

1 Introduction

Marine protected areas (MPAs) are areas placed in the ocean or in a water body where human activities are restricted to varying degrees and are often established with multiple objectives in mind such as ecosystem protection, sustainable use of the natural resource, food security, economic development, etc (O'Leary et al., 2018). There is a continuing dialogue between conservationists and fishers on creating a common understanding of the conservation objectives of MPAs and their business contributions (Westlund et al., 2017). For many fishers and aquaculture producers, MPAs are viewed as places where no farming is allowed, no-take and no-use zones, which is a misconception. The International Union for the Conservation of Nature (IUCN) has defined six categories of MPAs where the two most commonly applied types allow some aquaculture activities (Le Gouvello et al., 2017). For this study, we consider the no-take multi-purpose MPAs that allow aquaculture. This type of MPAs aims to preserve biodiversity and enhance a sustainable economy by managing related impacts and synergies. During the recent decades, around 85% of the world's fisheries are either being fished at full capacity or already more exploited (Le Gouvello et al., 2017; Sampantamit et al., 2020). Global fishery stocks are declining rapidly and are no longer capable of producing a sustainable amount due to over-fishing and habitat degradation. Thus, aquaculture gradually becomes an option to meet the shortfall and the increasing demand for fish (Sampantamit et al., 2020; Tewabe, 2015; Armstrong et al., 2016; Le Gouvello et al., 2017).

Aquaculture continues to grow at a faster rate than other agricultural sectors, but growth would not be sustainable if planning and management are not improved significantly (Hai et al., 2018). The performance of the fisheries mainly relies on strategic management and regulatory mechanisms that integrate the biological and ecological behavior of the resource in space and time with economic factors (Anderson and Seijo, 2010). In this research, we will investigate the advantage of developing aquaculture farms in multiple-use MPAs. We consider the case where there is an existing multiple-use MPAs and a call to develop aquaculture within the area or a creation of new multiple-use MPAs with aquaculture operations. Fish cultivation in MPAs must take into account the demand of the

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community and compatibility with the management objectives of the MPAs and free fishing area activities (Le Gouvello et al., 2017). MPAs under certain conditions and circumstances may be preferable for fish cultivation compared to open-access areas (Le Gouvello et al., 2017). In this situation, the type of aquaculture systems, rate of expansion, and the maximum optimal size of the area in the MPAs dedicated to fish farming would need careful selection. Management guidelines would have to account for the changes in biological and economic factors. For the rest of the paper, MPAs refer to multi-use MPAs.

One of the main factors in evaluating the effectiveness of MPAs on capture fisheries and conservation includes activities outside MPAs (Westlund et al., 2017). Most fishery researchers consider one of the following assumptions. First, some ecology-based papers focus on the assumption that all fish outside the MPAs, up to the maximum sustainable yield, would be harvested by using the available maximum effort (Cabral et al., 2019). The second common approach assumes MPAs allow fixed fishing harvest rates based on quota or effort-limiting policy (Akpalu and Bitew, 2014). The third approach, the one in which we use for this paper assumes the formation of MPAs with aquaculture activities as part of an optimized fishery management strategy (Le Gouvello et al., 2017). This plan still would be overly optimistic in the presence of poor fishery management. Considering the optimal size of MPAs, Cabral et al. (2019) developed a bioeconomic model of a fishery under open access, which is meant to represent a broad class of fisheries in developing countries. In that setting, they derived the MPAs size that maximizes food security (catch), recognizing that individual fishers respond to real-time economic conditions. On the other hand, the research conducted by Akpalu and Bitew (2014) studies the use of marine reserves as a management tool for minimizing the negative effects of fishing in areas where the species are biologically diverse. On resource allocation, Pichika and Zawka (2019) studied a harvesting effort for renewable resources in the presence of environmental pollution. The research showed the stock of pollution was assumed to affect both the saturation level and intrinsic per capita growth rate of the resource. In our paper, like Akpalu and Bitew (2018) and Pichika and Zawka (2019), we consider the negative externalities to the environment due to fishing and aquaculture activities. Moreover, we assume aquaculture activities are solely implemented in MPAs. Then we determine the optimal aquaculture size that can be implemented in an MPAs relative to its optimal size and optimal harvest effort outside the reserve. This dictates the coexistence of the MPAs, aquaculture production, and open-access fishery that preserves the environment and species diversity as well as supports economic development.

The remainder of the paper is organized as follows. In the next section, we set up deterministic dynamic equations of the stock of fish in and outside the MPAs without aquaculture and determine the Maximum Sustainable Yield (MSY) and optimal harvest with corresponding effort levels. In Section 3, we extend the model further by including aquaculture in the MPAs. In both cases, we analyze the control version of the models and find steady-state optimal solutions numerically for both the control and state variables. Section 4 presents the discussion and Section 5 concludes the paper.

2 Dynamic models and optimal management with MPAs

In this section, we describe the dynamics of capture fishery where fishing activities affect the habitat. Then we designate multiple-use MPAs for the purpose of protecting the environment and restocking the fish population in the fishing area. We also consider the control version of the model to determine the optimal size of the MPAs.

2.1 Capture fishery problem without MPAs

For this study, we start with the conventional ecological transition model given by

$$\frac{dX}{dt} = r\left(1 - \frac{X}{K}\right)X - \sigma EX, \tag{1}$$

where X is the size of the fish stock, r is the natural growth rate, K is the carrying capacity of the fishing environment, and σEX is the harvest, where σ is the catchability coefficient and E is the effort.

Recent studies show habitat degradation is more responsible for the sharp decline of the stock than overfishing (Tewabe, 2015; Kahui et al., 2016). Foley et al. (2012) studied the influence of habitat on intrinsic growth and carrying capacity. Armstrong et al. (2016) analyzed the consequences of exogenous habitat loss and the disproportionate impact of habitat degradation upon profits when the habitat is at a low level. Pichika and Zawaka (2019) formulated a dynamic equation of the stock where the growth rate of the population is affected by pollution. Following their models, we extend the above dynamic equation, equation (1), further by assuming that harvesting activity or effort negatively affects the fishing ground or the habitat, and hence the growth rate of the stock. Let its impact increases linearly with E. Therefore, we set up the dynamic equation for the fish stock as

$$\frac{dX}{dt} = (r_0 - \varepsilon_1 E)X\left(1 - \frac{X}{K}\right) - \sigma EX, \tag{2}$$

where $0 \leq r_0 - \varepsilon_1 E \leq r$, r_0 is the measure of the current growth rate, and $\varepsilon_1 \geq 0$ is the conversion factor of fishing effort level, E, to its impact on the growth rate.

To align the transition equation with our density-based analysis in the next section, we divide both sides of equation (2) by K and rewrite the equation as

$$\frac{dx}{dt} = (r_0 - \varepsilon_1 E)x(1 - x) - \sigma Ex, \tag{3}$$

where $x = \frac{X}{K}$ is the density of the stock.

2.1.1 Steady state analysis

The trivial equilibrium $(x = 0)$ is unstable, and the nontrivial equilibrium $\left(x = 1 - \frac{\sigma E}{r_0 - \varepsilon_1 E}\right)$ is stable whenever $r_0 - \varepsilon_1 E > 0$. The corresponding sustainable harvest (yield) function for each level of effort, $h_s(E)$, is given by:

Table 1. Parameters and their values used for numerical simulations.

Parameter	Description	Value
r_0	Growth rate parameter	1.8
σ	Catchability coefficient per unit effort	0.015
c	Cost per unit effort per unit carrying capacity for wild-catch	0.005
δ	Positive social discount rate	0.05
p	Per unit price in US dollars	15
d_1	Dispersion parameter	0.1

$$h_s(E) = \sigma x_s(E)E = \sigma E\left(1 - \frac{\sigma E}{r_0 - \varepsilon_1 E}\right). \tag{4}$$

The maximum sustainable yield (MSY) is attained at the effort level E where $\frac{\partial h_s(E)}{\partial E} = 0$. That is at

$$E_{\mathrm{MSY}} = r_0\left(\frac{(\varepsilon_1 + \sigma) - \sqrt{\sigma(\varepsilon_1 + \sigma)}}{\varepsilon_1(\varepsilon_1 + \sigma)}\right).$$

Substituting E_{MSY} into equation (4), we get

$$MSY(\cdot) = -\frac{\sigma\left(\sigma - \sqrt{\sigma(\varepsilon_1 + \sigma)}\right)\left((\varepsilon_1 + \sigma) - \sqrt{\sigma(\varepsilon_1 + \sigma)}\right)}{\varepsilon_1 2\sqrt{\sigma(\varepsilon_1 + \sigma)}.}$$

The sustainable profit is given by

$$\Pi(E) = ph_s(E) - cE,$$

and the maximum is attained at an effort level where $\frac{\partial \Pi(E)}{\partial E} = 0$, which implies

$$E^*() = r_0\left(\frac{p\sigma(\varepsilon_1 + \sigma) - \varepsilon_1 c - \sqrt{p\sigma^2(p\sigma(\varepsilon_1 + \sigma) - \varepsilon_1 c)}}{\varepsilon_1(p\sigma(\varepsilon_1 + \sigma) - \varepsilon_1 c)}\right), \tag{5}$$

provided $p\sigma(\_1 + \sigma) \geq \_1 c$, where p is the unit price and c is the cost of fishing per unit effort per unit carrying capacity. While the maximum sustainable yield reflects the potential of the fishing area, the maximum profit is the gap between the revenue and cost.

It may be noted that it is difficult to have the values of the parameters including the conversion factors involved in the model based on real-world observations. Some of the values are taken from Akpalu and Bitew (2018) and we choose reasonable values for other parameters based on the conditions we impose on the model for the purpose of numerical illustrations.

Using the values from Table 1 the maximum sustainable yield is MSY = 0.342945 attained at effort level $E_{\mathrm{MSY}} = 40.5726$. The maximum sustainable profit is attained at a lower optimal effort, $E^* = 39.9488$. Moreover, E^* declines with ε_1, permitting less fishing when the fishing impact is higher (see Fig. 1).

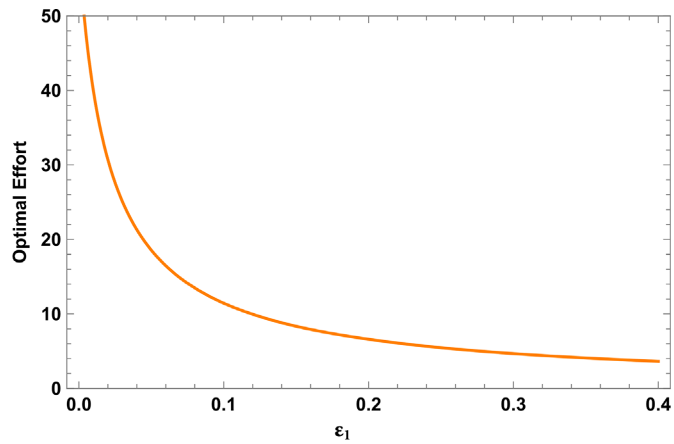

Fig. 1. The optimal effort as a function of the level of fishing activities impact on the growth rate.

2.2 Capture fishery with MPAs problem

Yamazaki et al. (2015) and Akpalu and Bitew (2014) studied harvest and/or effort control rules and no-take marine reserves as effective management tools for rebuilding depleted fish stocks and averting the collapse of fisheries. Following their formulation let M be part of the carrying capacity set aside for marine protected areas (MPAs) from the total capacity, K. Then the carrying capacity of the remaining area, the fishing area, becomes 1–m, where $m = \frac{M}{K} < 1$. The reserved environment gradually improves and starts positively impacting the growth of the stock of fish in the MPAs. We assume that the current growth rate increases with the size of m. Moreover, we consider that the density of fish in the MPAs over time becomes higher than the density of the stock outside the MPAs because of the no-take policy. Therefore, it initiates a density-based net influx of fish into the fishing ground, $d_1\frac{y}{m}$, where y is the density of fish in the marine protected area, $y = \frac{Y}{K}$, and d_1 is the dispersion rate (Akpalu and Bitew, 2014). Under these assumptions, we describe the dynamic equation of the stock in the MPAs as:

$$\frac{dy}{dt} = (r_0 + \varepsilon_2 m)y\left(1 - \frac{y}{m}\right) - d_1\frac{y}{m}, \tag{6}$$

where $0 \leq r_0 + \varepsilon_2 m \leq r$ and ε_2 is the measure of the positive impact of the MPAs on the growth rate of the fish in the area, and the stock dynamics outside the MPAs becomes[1]

$$\frac{dx}{dt} = (r_0 - \varepsilon_1 E)\left(1 - \frac{x}{1-m}\right)x + d_1\frac{y}{m} - \sigma Ex. \tag{7}$$

2.2.1 Steady state analysis

The equilibrium or steady state solution for the above system, equations (6) and (7), is

$$y_s(m) = m - \frac{d_1}{(r_0 + \varepsilon_2 m)} \geq 0, \tag{8}$$

$$\text{whenever } m(r_0 + \varepsilon_2 m) - d_1 \geq 0,$$

[1] If $m \to 1$ the equilibrium stock size $x^x(E,m) \to 0$ and the corresponding yield $h_x(E,m) \to 0$ implying there will be no human harvest.

and

$$x_s(E,m) = \frac{(1-m)\left(r_0 - \varepsilon_1 E - \sigma E \pm \sqrt{\frac{4d_1(r_0-\varepsilon_1 E)(m(r_0+\varepsilon_2 m)-d_1)}{(1-m)m(r_0+\varepsilon_2 m)} + (r_0 - (\varepsilon_1+\sigma)E)^2}\right)}{2(r_0-\varepsilon_1 E)}. \tag{9}$$

The positive steady state-state stock outside the reserve then is

See eq. (10a) Below

provided $r_0 - (\varepsilon_1 + \sigma)E > 0$

Substituting the steady-state solution $x_s(E, m)$ in the harvest function, the sustainable yield function is

See eq. (10b) Below

Then we solve the equation $\frac{\partial h_s(E,m)}{\partial E} = 0$ to determine the maximum sustainable effort, E_{MSY}, that corresponds to the MSY in terms of the parameters. Since the expression for the solution is very long, for numerical exposition, we substitute the values included in Table 1 and a reserve size, $m = 0.06$, to find maximum sustainable effort, $E_{MSY} = 42.6252$ with maximum sustainable yield $MSY = 0.337241$. Compared to this effort level, the maximum profit is attained at an effort level E such that

$$\frac{\partial \Pi(E,m)}{\partial E} = 0, \tag{11}$$

where $\Pi(E, m) = ph_s(E, m) - cE$ is the net profit at time t, for unit price, p, and the cost per unit effort per unit carrying capacity, c. The analytic solution for equation (11) is very long and complicated. For numerical illustration, we use values given in Table 1 and $m = 0.06$ to find the optimal effort level, $E^* = 41.897$ with corresponding optimal harvest $h^* = 0.337119$. Observe that the optimal effort and harvest are lower than the E_{MSY} and MSY. We can verify that the optimal solutions are stable for appropriate values of the parameters. For example, using the values given in Table 1 and assigning $m = 0.06$ and $E = 41.897$, and linearizing equations (6) and (7) about the optimal solution $x^* = 0.5364$ and $y^* = 0.0062$, we find the two roots corresponding to the characteristic polynomial, $r_1 = -0.7746$ and $r_2 = -0.1933$, which are real and negative. Therefore, (x^*, y^*) is a stable equilibrium.

2.2.2 Optimal capture fish management with MPAs

Suppose the sole manager of the fishery decides on harvesting effort, E, and the size of the MPAs, m. For simplicity, we assume that the unit price, p, and cost per unit effort per unit carrying capacity, c, are constants. If all future costs and benefits are discounted at a positive social discount rate of δ, the objective of the manager is to maximize the overall present value of the discounted stream of surpluses (which we denote by $MP(E, m)$, i.e.,

$$\max MP(E,m) = \max_{E,m} \int_0^\infty (p\sigma x - c)E_e{-}\delta t dt$$

subject to the dynamic equations (6) and (7).

The current value Hamiltonian corresponding to this problem is given by

$$\begin{aligned} H(\cdot) &= (p\sigma x - c)E \\ &+ \lambda_1\left((r_0 - \varepsilon_1 E)x\left(1 - \frac{x}{1-m}\right) - \sigma E x + \frac{d_1 y}{m}\right) \\ &+ \lambda_2\left((r_0 + \varepsilon_2 m)y\left(1 - \frac{y}{m}\right) - \frac{d_1 y}{m}\right), \end{aligned} \tag{12}$$

where λ_1 and λ_2 are the shadow value of the stock in the fishing area and in the MPAs, respectively.

Using equations (6) and (7), the first conditions for optimality with respect to the flow variables, E and m, and the co-state equations to each state variable, at the steady state we get the following system of non-linear equations

$$(r_0 - \varepsilon_1 E)x\left(1 - \frac{x}{1-m}\right) + \frac{d_1 y}{m} - \sigma E x = 0, \tag{13}$$

$$(r_0 + \varepsilon_2 m)y\left(1 - \frac{y}{m}\right) - \frac{d_1 y}{m} = 0, \tag{14}$$

$$-c + \lambda_1\left(-\varepsilon_1 x\left(1 - \frac{x}{1-m}\right) - \sigma x\right) + p\sigma x = 0, \tag{15}$$

$$\begin{aligned} &\lambda_1\left(-\frac{(r_0 - \varepsilon_1 E)x^2}{(1-m)^2} - \frac{d_1 y}{m^2}\right) \\ &+ \lambda_2\left(\frac{(r_0 + \varepsilon_2 m)y^2}{m^2} + \varepsilon_2 y\left(1 - \frac{y}{m}\right) + \frac{d_1 y}{m^2}\right) = 0, \end{aligned} \tag{16}$$

$$x_s(E,m) = \frac{(1-m)\left(r_0 - \varepsilon_1 E - E\sigma + \sqrt{\frac{4d_1(r_0-\varepsilon_1 E)(m(r_0+\varepsilon_2 m)-d_1)}{(1-m)m(r_0+\varepsilon_2 m)} + (r_0 - (\varepsilon_1+\sigma)E)^2}\right)}{2(r_0-\varepsilon_1 E)}, \tag{10a}$$

$$h_s(E,m) = \sigma E\left(\frac{(1-m)\left(r_0 - \varepsilon_1 E - \sigma E + \sqrt{\frac{4d_1(r_0-\varepsilon_1 E)(m(r_0+\varepsilon_2 m)-d_1)}{(1-m)m(r_0+\varepsilon_2 m)} + (r_0 - (\varepsilon_1+\sigma)E)^2}\right)}{2(r_0-\varepsilon_1 E)}\right). \tag{10b}$$

Table 2. Capture fish and MPAs numerical results and sensitivity analysis.

ε_1	ε_2	d_1	x^*	y^*	E^*	m^*	h^*
0.01	1	.1	0.50803	0.05468	45.3819	0.10712	0.345826
0.02	**1**	.1	0.54678	0.05720	34.7805	0.10957	**0.28526**
0.01	**2**	.1	0.50846	0.05742	45.6083	0.10707	**0.34785**
0.01	1	.08	0.51808	0.04328	44.3795	0.08570	**0.34488**

optimal capture stock size, y^* = optimal MPAs area stock size, E^* = optimal effort and h^* = optimal harvest, and m^* = optimal MPAs size.

$$\delta\lambda_1 = \lambda_1\left((r_0-\varepsilon_1 E)\left(1-\frac{x}{1-m}\right)-\frac{(r_0-\varepsilon_1)Ex}{1-m}-\sigma E\right) + \sigma pE, \tag{17}$$

$$\delta\lambda_2 = \frac{d_1\lambda_1}{m} + \lambda_2\left((r_0+\varepsilon_2 m)\left(1-\frac{y}{m}\right)-\frac{(r_0+\varepsilon_2 m)y}{m}-\frac{d_1}{m}\right). \tag{18}$$

By assigning the values for the parameters from Table 1 and the combination of the conversion factors given in Table 2, we solve the above system of equations using MATHEMATICA 13.0$^{\text{TM}}$ for the steady-state optimal solutions, the optimal effort and optimal stock including the optimal harvest summarized in Table 2. We also perform a sensitivity analysis of the model with respect to the conversion factors (see Tab. 2).

From Table 2 we observe that when the impact of harvesting effort on the environment becomes more severe, the optimal steady-state effort decreases and the size of the MPAs slightly increases. Moreover, if the benefit of the MPAs to the reserved environment (to the growth rate) is better than expected (i.e., when ε_2 increases), the optimal steady-state size of the MPAs decreases, and the effort level increases. And if the dispersion rate is lower than predicted, we must reduce the size of the MPAs and increase effort.

3 Capture fishery and MPAs with aquaculture problem

In this section, we consider the case where we introduce aquaculture within the MPAs. Although aquaculture activity negatively impacts the MPAs, compared to the fishing ground, we assume that over time the stock in the MPAs will still have a better density. The size of the aquaculture must be small compared to the size of the MPAs so that the MPAs serve their intended purposes (i.e., environmental and ecological protections). We will find an optimal aquaculture size relative to the optimal size of the MPAs that maximizes the overall economic and ecological benefits.

As mentioned earlier, like the fishing activities outside the MPAs, farming activities inside the MPAs affects the reserved environment and consequently lower the per capita growth rate of the stock in the area. We assume the extent of the impact is directly proportional to the aquaculture size, A, hence the carrying capacity of the MPAs becomes $m-\gamma a$, where $a=\frac{A}{K}<m=\frac{M}{K}$ and γ is a conversion factor. In addition, the spillover factor depends on the relative size of a, that is on $1-\frac{a}{m}$. Therefore, we set up the biomass dynamics in and outside the MPAs as

$$\frac{dy}{dt}=(r_0+\varepsilon_2 m)\left(1-\frac{y}{m-\gamma a}\right)y-d_2\left(1-\frac{a}{m}\right)\frac{y}{m}, \tag{19}$$

and

$$\frac{dx}{dt}=(r_0-\varepsilon_1 E)\left(1-\frac{x}{1-m}\right)x-\sigma Ex + d_2\left(1-\frac{a}{m}\right)\frac{y}{m}. \tag{20}$$

The main purpose of aquaculture production in MPAs is to meet societal demands while balancing economic benefits and environmental protection. Moreover, aquaculture is more productive in MPAs than in open-access areas because of the clean environment or water quality (Le Gouvello et al., 2017). Aquaculture helps to redirect and ease the pressure on the fishing environment caused partly by the allocation of some areas for MPAs and by intense fishing activity due to the decline of the stock. We assume that aquaculture is developed solely in MPAs and the expansion rate of aquaculture depends on the wild-catch harvest level and the proportion of the area in the MPAs dedicated to aquaculture. Hence, we set up the aquaculture dynamics as

$$\frac{da}{dt}=\left(v-\frac{a}{m}\right)(1-\sigma Ex), \tag{21}$$

where v is a control variable related to the expansion rate and $v \geq \frac{a}{m}$.

3.1 Steady state analysis

The non-negative equilibrium solution/point for equations (19) through (21), (a_s, y_s, x_s), is

$$a_s(m,v)=vm,$$

$$y_s(m,v)=\frac{(1-\gamma v)(m(r_0+\varepsilon_2 m))-d_2(1-v))}{r_0+\varepsilon_2 m},$$

provided $m(r_0+\varepsilon_2 m))>d_2\,(1-v)$, and

$$x_s(E,m,v) = \frac{(1-m)\left(r_0 - \varepsilon_1 E - \sigma E + \sqrt{(r_0 - \varepsilon_1 E - \sigma E)^2 - \frac{4d_2(1-v)(1-\gamma v)(r_0-\varepsilon_1 E)(d_2(1-v)-m(r_0+\varepsilon_2 m))}{(1-m)m(r_0+\varepsilon_2 m)}}\right)}{2(r_0 - \varepsilon_1 E)}.$$

See eq. Above

Substituting x_s (E,m,v) into the harvest function, we get the equilibrium harvest (yield) function:

$$h_s(E,m,v) = \sigma E x_s(E,m,v). \tag{22}$$

By substituting the values for σ, r_0, ε_1, ε_2, γ, and d_2 from Table 1 in (22) and assigning a reserve size, $m=0.06$, and $v=0.47$, and solving the equation $\frac{\partial h_s(E,m,v)}{\partial E} = 0$, the maximum sustainable effort, $E_{\text{MSY}}=42.7769$ with $MSY=0.338603$. And the maximum profit is attained at effort level E such that $\frac{\partial(ph_s(E,m,v)-cE)}{\partial E} = 0$, where p is the per unit price and c cost per unit harvest/carrying capacity. For p = \$15 per kilogram, c= \$0.005, and the values from Table 1 the maximum profit attained at optimal effort level $E^*=41.6757$ with optimal harvest, $h^*=0.338327$. Moreover, the Jacobian matrix that corresponds to the dynamical system, at the optimal solution, ($a^*=0.0329$, $x^*=0.5412$, $y^*=0.0134$),

$$J = \begin{pmatrix} -0.796326 & 1.10685 & -0.443866 \\ 0 & -0.0832308 & 0.434674 \\ 0 & 0 & -13.8825 \end{pmatrix}$$

has all negative eigenvalues

$$Eigenvalues(J) = \begin{pmatrix} -13.8825 \\ -0.796326 \\ -0.0832308 \end{pmatrix}$$

Therefore, the optimal solution is stable.

3.1.1. Comparison of the three sustainable yield trajectories

In Figure 2, we compare the trajectories of the steady-state yield functions with respect to the level of fishing effort in the three scenarios.

From Figure 2, even though the maximum sustainable yield is lower after the implementation of the MPAs, the fish species are protected from a possible collapse caused by overfishing and other factors. Moreover, we observe a further reduction of the maximum sustainable yield after the development of aquaculture in the MPAs due to an adjustment of the effort level (less effort) outside the MPAs.

3.2 Optimal capture fish effort and optimal aquaculture size within the MPAs

Suppose the production function for aquaculture is $Z(A)=P_a A$ and the cost of production is quadratic $\Phi(A)= c_a(P_a)^2 A$, where P_a is per unit area production of farmed fish and c_a is a cost parameter (Mykoniatis and Ready, 2016). We assume that buyers cannot distinguish between a species that is farmed or caught in the wild. Let the unit price of both fish be constant, p. Hence the net profit from aquaculture at time t is $pP_a a - c_a(P_a)^2 a$. In our analysis, we include a separate one-time cost of acquiring extra units of the MPAs. Let the cost per unit area/carrying capacity be c_3.

We can rewrite the production and cost functions in terms of a as $z(a)=P_a a$ and $\varphi(a)=c_a(P_a)^2 a$, where $z(a) = \frac{Z(A)}{K}$ and $\varphi(a) = \frac{\Phi(A)}{K}$, K is the carrying capacity. Then the optimization problem is

$$\max MPAs(E,m,v) = \max_{E,m,v} \int_0^\infty ((p\sigma x - c)E + pz(a) - \varphi(a) - c_3\left(v - \frac{a}{m}\right)(1-\sigma E x))e^{-\delta t}dt, \tag{23}$$

subject to the dynamic equations (19), (20), and (21).

The current value Hamiltonian for this problem is

$$H(\cdot) = B(x,a,E,m) + \lambda_1\left((r_0 - \varepsilon_1 E)\left(1 - \tfrac{x}{1-m}\right)x - \sigma E x + d_2\left(1 - \tfrac{a}{m}\right)\tfrac{y}{m}\right) + \lambda_2\left((r_0 + \varepsilon_2 m)y\left(1 - \frac{y}{m - \gamma a}\right) - d_2\left(1 - \frac{a}{m}\right)\frac{y}{m}\right) + \lambda_3\left(v - \frac{a}{m}\right)(1 - \sigma E x), \tag{24}$$

where $B(x,a,E,m) = (p\sigma x - c)E + pc_1 a - c_2 a - c_3\left(v - \frac{a}{m}\right)(1 - \sigma E x)$. where $c_1=P_a$ production per unit aquaculture area and $c_2=c_a(P_a)^2$ is the cost per unit area, and λ_1, λ_2, and λ_3 are the shadow values of stock outside and inside the MPAs and the aquaculture area, respectively.

Using the first conditions for optimality with respect to the flow variables, E, m, and v:

$$-c + \sigma x\left((c_3 - \lambda_3)\left(v - \frac{a}{m}\right) - \lambda_1 + p\right) - \frac{\varepsilon_1 \lambda_1 x(1 - m - x)}{1 - m} = 0 \tag{25}$$

$$\lambda_1\left(\frac{d_2 y(2a - m)}{m^3} - \frac{x^2(r_0 - E\varepsilon_1)}{(m-1)^2}\right) + \lambda_2 y\left(\frac{d_2(m - 2a)}{m^3} + \frac{\varepsilon_2\left(m^2 + r_0 y - 2a\gamma m + a\gamma(a\gamma + y)\right)}{(m - a\gamma)^2}\right)$$

$$-\frac{a(c_3 + \lambda_3)(1 - \sigma E x)}{m^2} = 0 \tag{26}$$

$$(c_3 - \lambda_3)(1 - \sigma E x) = 0 \tag{27}$$

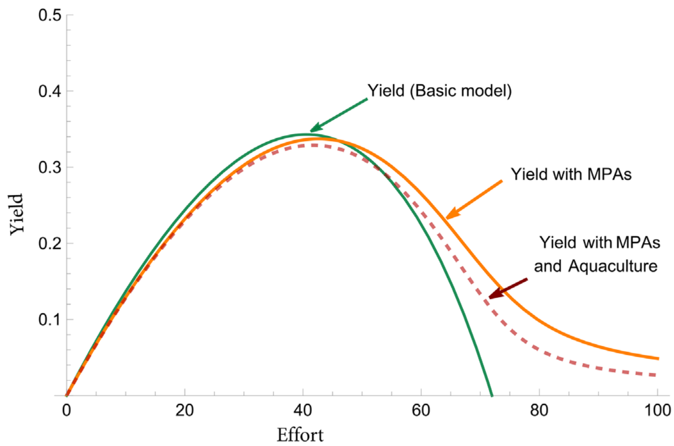

Fig. 2. The yield versus effort curve for the basic model, with MPAs, and with MPAs and aquaculture in MPAs.

and the co-state equations to each state variable:

$$-\frac{d\lambda_1}{dt}+\delta\lambda_1=\sigma E\left((c_3-\lambda_3)\left(v-\frac{a}{m}\right)-\lambda_1+p\right)-\frac{\varepsilon_1\lambda_1(1-m-2x)E}{1-m}+r_0\left(\lambda_1-\frac{2\lambda_1 x}{1-m}\right)\tag{28}$$

$$-\frac{d\lambda_2}{dt}+\delta\lambda_2=\frac{d_2(m-a)(\lambda_1-\lambda_2)}{m^2}+\frac{\lambda_2(r_0+\varepsilon_2 m)(m-2y-\gamma a)}{m-\gamma a}\tag{29}$$

$$-\frac{d\lambda_3}{dt}+\delta\lambda_3=-c_2+c_1 p-\frac{\gamma\lambda_2 y^2(r_0+\varepsilon_2 m)}{(m-\gamma a)^2}-\frac{(c_3-\lambda_3)(1-\sigma Ex)}{m}+\frac{d_2 y(\lambda_2-\lambda_1)}{m^2}\tag{30}$$

At the steady state, the state and co-state variables including the control variables satisfy the following equations: $\frac{dx}{dt}=\frac{dy}{dt}=\frac{da}{dt}=0$, $\frac{\lambda_1}{dt}=\frac{\lambda_2}{dt}=\frac{\lambda_3}{dt}=0$ and equations (23) to (25). This is a system of nine non-linear equations and the formulas for the analytic solutions in terms of the parameters are beyond comprehension.

3.1.2 Numerical illustration

For the purpose of numerical solutions, we assign the values given in Table 3 and the combination of the conversion factors given in Table 4. Then using MATHEMATICA 13.0™, we find the corresponding optimal solutions presented in Table 4. We also perform a sensitivity analysis to see the effects of the change in the conversion factors and dispersion rate on the steady-state solutions.

From the steady-state optimal numerical solutions summarized in Table 4

i) As the impact of effort on the fishing environment worsens, the size of the MPAs, as well as the effort, must be lowered to ease the pressure on the wild capture fish. This creates a scarcity of wild fish in the market. However, the optimal management strategy implies that the decline in the supply of wild fish can be matched with an expansion of aquaculture production.

ii) If the environmental recovery rate of the reserved area is better than expected or the dispersion rate is lower, the MPAs could be set to a smaller size, and the effort level in the fishing area can be relaxed.

iii) If the impact of fishing farming on the growth rate of the fish stock in MPAs is more severe, the reserve area should be expanded and we reduce the aquaculture activity and lower the effort level outside MPAs.

3.3 Transition dynamics and stability

3.3.1 Transition dynamics

Because of the non-linear nature of the functional forms of the equations used in the dynamic analysis and the number of equations, it is not easy to find analytic solutions. Therefore, we use the fourth-order Runge-Kutta forward-backward sweep method to solve the system of equations numerically. First, we approximate the state equations, (19) through (21), by first-order forward difference and the corresponding co-state equations, (28) through (30), by first-order backward difference equations. Then by substituting the values of the parameters given in Table 3, using initial values, $x(0)=0.27$, $y(0)=0.006$, and $a(0)=0$, and using a guess for optimal control say the steady-state values, we solve the state equations forward for the discrete-time interval of $[0, t_f]$ partitioned into n parts using a time step h such that $t_f=hn$. Then using the state values and the transversality condition at t_f, we solve the co-state equations backward. After each forward-backward computation, we update the control values using the state and co-state values and repeat the process until the control values become sufficiently close. The accuracy or convergence of the iterative method is based on Hackbusch (1978). Figure 3 displays the trajectories of the numerical solutions generated using Hertmite interpolation of order 3. Note that for all combinations of the conversion factors and the dispersion rate in Table 3, the trend is the same except they converge to different state and control values.

3.3.2 Stability

Following the computation of the optimal trajectories of the system, it is necessary to analyze the reaction of the system when an unexpected change or shock occurs due to exogenous factors. In particular, to investigate the stability of the equilibrium, we linearized the equations (19) through (21) and (28) through (30) around the equilibrium points/values. Then using the parameter values and the corresponding steady-state values of the variables x, y, a, λ_1, λ_2, λ_3, E, m, and v, the Jacobian matrix of the model is

$$\mathbf{J} = \begin{pmatrix} -0.796326 & 1.10691 & -0.443855 & 0 & 0 & 0 \\ 0 & -0.740977 & 0.146733 & 0 & 0 & 0 \\ 0 & 0 & -13.8827 & 0 & 0 & 0 \\ 32.2786 & 0 & 0.0131046 & 0.846326 & 0 & 0 \\ 0 & 2266.31 & 715.027 & -1.10691 & 0.790977 & 0 \\ 0.0131046 & 715.027 & 364.401 & 0.443855 & -0.146733 & 13.9327 \end{pmatrix}$$

The corresponding eigenvalues are

$$\textbf{Eigenvalues}\ (\mathbf{J}) = \begin{pmatrix} 13.9327 \\ -13.8827 \\ 0.846326 \\ -0.796326 \\ 0.790977 \\ -0.740977 \end{pmatrix}$$

The real parts of the eigenvalues have positive and negative values. The signs of the eigenvalues are consistent across all the other combinations of the values of the parameters used for the empirical analysis. As a result, the control version of the system is unstable. It implies that any deviation from the steady state equilibrium due to say changes in the price of the fish, the cost per unit effort, or the cost of aquaculture production will lead to further departure from the equilibrium. Since all the eigenvalues are non-zero real, the equilibrium is hyperbolic (i.e. it is robust, a small perturbation displaces the equilibrium by a small amount). This shows that economic factors play a significant role in the profitability and stability of the sector. Especially, in developing countries, the relationship between production cost and revenue determines the feasibility of aquaculture.

4 Discussion

The effectiveness of the optimized management plan depends on the cooperation and information-sharing practices between the capture fishery and aquaculture managers. Moreover, in marine natural resource management, it is vital to understand the current status of the ecological environment before introducing a new management strategy. Then we plan accordingly toward achieving a sustainable system over time following the predefined fast-approaching path. For this reason, we solve the system of non-linear differential equations derived from the necessary conditions for optimality. We find the expansion rate of the aquaculture and hence the recovery rate of MPAs and the wild fishery stocks. We also perform sensitivity analysis to show the necessary adjustments we must make to the paths when the performance of the growth rate parameters and the impact of aquaculture is different from the predicted values. It may be noted that the values of the parameters and other values are not based on real observations. Therefore, the numerical results and the corresponding recovery and expansion rate curves should be considered from a qualitative point of view.

The optimal population growth curve in the reserved area, Figure 3, shows that the stock recovers rapidly while gradually restocking the fishing ground. We also observed that the implementation of aquaculture doesn't change the trend of the trajectories of the stocks and the MPAs still shield the stock (see Fig. 2). However, integrating aquaculture in marine reserves slows down the fishery rebuilding process inside and outside the reserved area. When the impact of aquaculture is higher than expected (i.e., when ε_2 increases), the recovery rate of the reserved area (hence the open access) converges to a lower optimal equilibrium (see Fig. 4).

If an unexpected shock, such as an oil spill and chemical contamination due to human activity, affects the open access (which can be interpreted as a higher-level impact of effort on the habitat, i.e., larger ε_1), the optimal effort will be reduced significantly. This helps to minimize the impact of the effort and facilitates the remediation process. The effort-limiting policy creates a scarcity of wild fish in the market. In this case, we can increase the size of the aquaculture to satisfy the demand for fish. We assumed earlier that consumers can't distinguish between wild catch and farmed fish (this is a common practice in most developing countries) and both are sold at the same price. Therefore, the decline in the wild catch can be matched with its substitute or alternative, farmed fish, by increasing aquaculture production.

As the dynamic optimization model implies, one of the management strategies is to lower the wild catch effort level if the intensity of its impacts on the habitat is higher than predicted. This restriction has implications for the fisheries community. In this situation, to divert the effort from open access and to ease the tension between the two sectors, the concerned group (the government) must encourage some of the fishers to get involved in the aquaculture industry. This can be done by prioritizing aquaculture permits for capture fishers and providing incentives, like monetary and technical support.

In addition to the optimized space allocation practices, the profitability and sustainability of the aquaculture sector depend on price to per-unit production and cost of production. By assumption, the net profit from aquaculture at time t is given by $\Pi(P_a) = pP_a a - c_a (P_a)^2 a$, where P_a is per unit production and c_a is a cost parameter (Mykoniatis and Ready, 2016). Therefore, the feasibility of the business is determined by the relation $\Pi(P_a) \geq 0 \Rightarrow pP_a a \geq c_a (P_a)^2 a$. Most of the time this relationship is not easy to achieve, especially at the beginning of the investment. Therefore, we suggest that investors must get incentives such as subsidized loans and tax relief until their revenue becomes larger than the cost of production. If investors are encouraged and take part in the aquaculture industry, in the long run, we can maintain a sustainable supply of fish at the same time support the economy by creating jobs.

Table 3. Parameters and their values used for numerical solutions.

Parameter	Description	Value
r_0	Growth rate parameter	1.8
$c_1 = P_a$	Aquaculture production per unit square meter in kg	44.6
c_a	Cost parameter for aquaculture that maximizes the net profit	0.1682
c_2	Cost parameter for aquaculture	6.3464
c_3	Cost of acquiring a square meter of aquaculture area in US dollars	1
σ	Catchability coefficient per unit effort	0.015
c	Cost per unit effort per unit carrying capacity for wild-catch	0.05
δ	Positive social discount rate	0.05
p	Per unit price in US dollars	15
d_2	Dispersion parameter	0.1

Table 4. Capture fishery, MPAs, and aquaculture numerical results and sensitivity analysis.

ε_1	ε_2	γ	d_2	x^*	y^*	a^*	E^*	m^*	h^*
.01	1	1	.1	0.53422	0.01018	0.02251	41.8339	0.04789	0.33473
.02	1	1	.1	0.57469	0.00860	0.02319	31.9525	0.04379	0.27544
.01	2	1	.1	0.53651	0.00846	0.02205	41.6916	0.04323	0.33552
.01	1	**2**	.1	0.53313	0.00417	0.01517	41.3825	0.04833	0.33093
.01	1	1	.08	0.53869	0.00849	0.01805	41.6713	0.0392	0.33672

x^* = optimal capture stock size, y^* = optimal stock in the MPAs, a^* = optimal aquaculture size, E^* = optimal effort, h^* = optimal harvest, and m^* = optimal MPAs size.

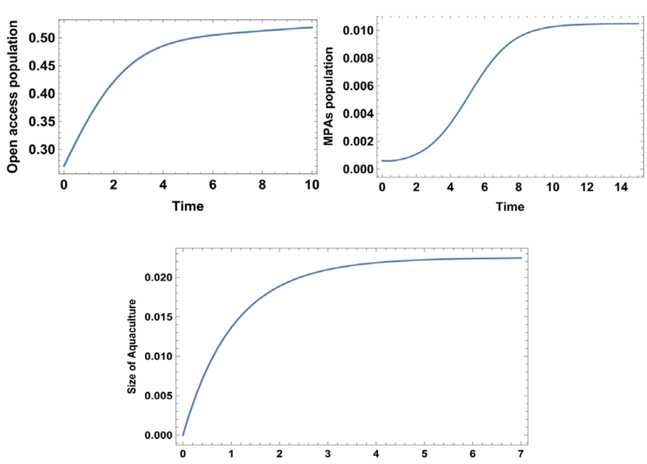

Fig. 3. The recovery rates of the open access and MPAs stocks and the expansion rate of the aquaculture within the reserve for the optimal reserve size, $m = 0.04789$.

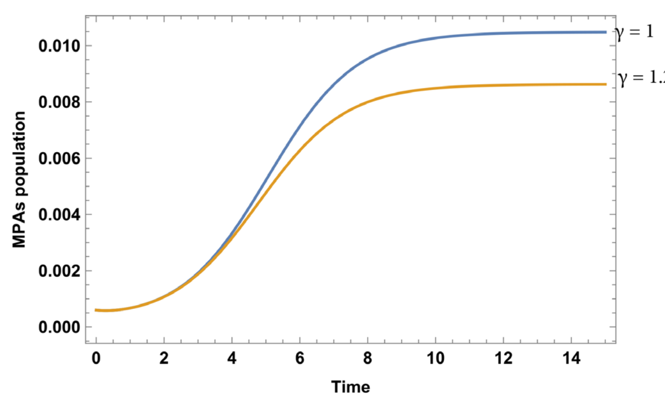

Fig. 4. The MPAs stock for $\gamma = 1$ and $\gamma = 1.2$ and $m = 0.04789$.

5 Conclusion

MPAs have an advantage in the conservation of the natural environment and help to protect and sustain the fish population. In addition, they have economic benefits if aquaculture production is operating in the area even though it has negative externalities. In this paper, we attempt to determine the optimal size of fish farming in MPAs based on the level of food security and the ecological status of the fishing environment by allocating the right amount of area from the open access areas toward MPAs. This helps in restocking the fishing area at the same time to protect the species of fish from possible collapse.

Le Gouvello et al. (2017) in their study investigated how can MPAs support aquaculture development, how should aquaculture activities support MPAs and how can negative interactions be minimized and greater common trust between fisheries can be achieved. They recommend that a set of principles must accompany the process of setting up aquaculture for reconciling aquaculture and MPAs. The establishment of an aquaculture farm in the MPAs requires a long-term growth and monitoring plan. Therefore, it is necessary to decide the appropriate size of the aquaculture in MPAs for poverty alleviation and food security of MPA's local communities while supporting the recovery process and advancing the resilience of the environment. The results obtained from our study will fill this gap and provide insight into the optimal sizes of multiple-use MPAs and aquaculture size relative to the size of MPAs. It also determines the optimal harvesting effort outside MPAs for sustainable fisheries management. To the best of our knowledge, this is the first dynamic bioeconomic model that considers aquaculture in MPAs and attempts to determine the optimal size of aquaculture relative to reserved areas and fishing efforts.

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