Aquat. Living Resour. 22, 127–133 (2009)
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DOI: 10.1051/alr/2009015
www.alr-journal.org

Aquatic
Living
Resources

Statistical estimation of mean values of fish stock indicators from trawl surveys

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Received 26 June 2008; Accepted 2 April 2009

Abstract – The mean or average value of a measure or indicator variable used to monitor a fish population can be defined in various ways, each with a correspondingly different statistical estimator for use in the context of a trawl survey. When, as is typical for many species, fish are heterogeneously clustered over space, then scooped in clusters from restricted localities using a trawl, these different estimators can produce different sample mean values with contrasting variations over time, possibly leading to different inferences about the fish population. Two mean parameters and their intuitive estimators, the mean "over fish", and the mean "over stations", are discussed and found to present contrasting statistical properties. A third estimator based on fitting a mixed model is proposed which has intermediate properties based on the within-haul correlation. The three estimators are applied illustratively to length data for cod caught in the North Sea by the English groundfish survey from 1992 to 2007. The time series of the mean over stations was smoothest, that for the mean over fish much more variable, and the mixed mean fell between in all years. Variance estimators derived from the fitted mixed model are also put forward. Estimates made from the example suggested that the mixed mean is most efficient. The type of estimator used for the mean should always be considered carefully and mentioned when reporting indicator studies.

Key words: Statistics / Mean over fish / Mean over stations / Trawl survey / Indicator / Mixed model

1 Introduction

The mean value over some defined geographic region of an indicator or measure describing fish caught during a trawl survey is commonly estimated either as an average over all fish caught, or as an average of the average values at each fishing station. I refer to these as the "mean over fish" and the "mean over stations", respectively. Typical examples of the measures I am referring to are body length, age, trophic level, and weight of stomach contents. Pennington and co-workers discuss the two estimators and their sampling variances in the context of trawl surveys and market sampling, pointing out that trawling catches clusters of fish, not random samples, and therefore that precision is generally much lower than a count of the fish measured would suggest (Pennington and Vølstad 1994; Pennington et al. 2002; Aanes and Pennington 2003; Anonymous 2005). There remains, however, scope for confusion about the two estimators because they correspond to variously named estimators in sampling textbooks, and because the nature of what is being estimated is not always explicit. Here, I attempt to throw light on both of these aspects with the intention of explaining why the formulation of the mean used in a study of indicators is important.

Secondly, I point out that the mean over fish is unaffected by the number of stations where the fish were caught provided that there was at least one, while the mean over stations makes no allowance for the number of fish caught at each station provided that at least one was caught. I argue that the two estimators lie at the opposite ends of a spectrum formed by a measure of clustering, the intra-haul correlation, and that a third, estimator based on it, may be preferable. It is referred to as the "mixed" mean because it requires a mixed linear model to be fitted. Formulae are derived and all three estimators are calculated for a set of survey data for lengths of North Sea cod for comparison and illustration.

For simplicity, only surveys having an approximately uniform density of fishing stations per unit area are considered. For stratified surveys with different densities in different strata, read "stratum" where "survey" is written; brief suggestions on stratum weighting when using different estimators of the mean are given in the concluding Comments section. Constant catchabilities are assumed throughout; i.e. no consideration is given here to variable fishing or catch sampling practices, to disturbances due to previous trawling, or to effects on catchabilities of depth, gear problems, time of day, substrate type, visibility, etc. These aspects are at least as important as the choice of estimators for the mean, but they are dealt with elsewhere (Anonymous 2004b, 2006). Concerning notation, formulae

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quoted from other publications have all been re-written to conform with the trawl-related notation adopted here.

2 Theory

2.1 The stock mean

Firstly, a fish stock can be considered as a population of N individuals, counted by j. With this approach, fish are the "sampling units" in the terminology of sampling theory. Then, if the individuals all have some attribute or measure, y, to be used as an indicator, the true mean value for the stock is defined as

$$\bar{Y}_{\text{pop}} = N^{-1} \sum_{j=1}^{N} y_j. \quad (1)$$

Secondly, the same stock can be considered in a geographical sense consistent with the geographic spread of fishing stations used by most trawl surveys. Suppose that the stock is distributed over a domain, D, divided into a totally inclusive grid of A small plots. Then plots are being treated as the sampling units. If there are n_i fish in the i'th plot, and the sum of the individual measures in the plot is y_i, then the stock mean can also be defined as

$$\bar{Y}_{\text{geog}} = \frac{\sum_{i=1}^{A} y_i}{\sum_{i=1}^{A} n_i} \quad (2)$$

$\bar{Y}_{\text{geog}}$ does not depend on the size or orientation of the plots used for its definition provided that the plots do not overlap or have gaps between them. Then $\bar{Y}_{\text{pop}} = \bar{Y}_{\text{geog}}$ regardless of the size or orientation of the plots. These two ways of conceptualising the true stock mean value imply different types of sampling unit and can complicate the choice of the best sampling formula for estimating the mean.

2.2 Trawl sampling

A mean is estimated without statistical bias when sampling units are selected from a population by simple random sampling (Thompson 1992). Thus to avoid bias with this sampling scheme, fish should be selected individually and at random from the stock to estimate $\bar{Y}_{\text{pop}}$, and geographic plots should be selected similarly to estimate $\bar{Y}_{\text{geog}}$. Both sampling approaches are problematic for trawl surveys.

Trawls do not select fish randomly because of their typically patchy distribution in the sea (Pennington et al. 2002). For example, one catch may contain predominantly small fish taken near a nursery ground, while another may contain large individuals from a favoured feeding area. In statistical terms, the fish within a catch are mutually dependent, or there is a high degree of "intra-cluster correlation" (Cochran 1977). Even if each fish could be caught independently in a practical way for a survey, their individual locations are unknown so an equal probability of selection cannot be assigned to every member of the population for the purpose of simple random sampling.

The trawl-towing paths followed at individual fishing stations can reasonably be equated with the "plots" referred to in the definition of $\bar{Y}_{\text{geog}}$. There can, however, be questions over untrawlable areas and whether the set of all feasible, candidate fishing stations adequately covers the stock domain without overlaps or gaps. Practical factors such as substrate type, tide, depth, and weather are all likely to affect the comparability of tow paths from one station to another (as is also relevant to many other types of analysis of trawl survey data).

2.3 The two estimators

Let y_{ij} represent the value of an attribute measured on the j'th fish caught at the i'th station, where now $i = 1, \ldots, n_{\text{stn}}$ and $j = 1, \ldots, n_i$. n_{stn} is the number of fishing stations where at least one fish was caught, and n_i is the number of fish caught at the i'th station with $\sum_{i=1}^{n_{\text{stn}}} n_i = n_{\text{fish}}$, the total number of fish caught on the survey. The two estimators of the stock mean are:

The mean *over fish*,

$$\bar{y}_{\text{fish}} = \frac{\sum_{i=1}^{n_{\text{stn}}} \sum_{j=1}^{n_i} y_{ij}}{n_{\text{fish}}}. \quad (3)$$

The mean *over stations*,

$$\bar{y}_{\text{stn}} = \frac{\sum_{i=1}^{n_{\text{stn}}} \bar{y}_i}{n_{\text{stn}}} = \frac{\sum_{i=1}^{n_{\text{stn}}} n_i^{-1} \sum_{j=1}^{n_i} y_{ij}}{n_{\text{stn}}}. \quad (4)$$

Various relationships of the mean over fish (3) to the two definitions of the stock means, (1) and (2) can be argued theoretically. Firstly, formula (3) is equivalent to the simple random sampling mean when fish are the sampling units. It is therefore statistically consistent for $\bar{Y}_{\text{pop}}$. Secondly, it is also a weighted average over stations because those stations yielding the most fish have the most influence on the estimate. This view of $\bar{y}_{\text{fish}}$, in contrast, implies that stations are the sampling units, as for $\bar{Y}_{\text{geog}}$. Thirdly, Pennington and Vølstad (1994) refer to a formula equivalent to $\bar{y}_{\text{fish}}$ as a ratio estimator (Cochran 1977; Thompson 1992) with the observed abundance, n_i, and the sum of all attribute values, $\sum_{j=1}^{n_i} y_{ij}$, being twin attributes of each station, treated as the sampling unit. This view allows for the randomness of the n_i when cluster sampling with a trawl. Fourthly, the formula for $\bar{y}_{\text{fish}}$ approximates that for single-stage cluster samples with unequal cluster sizes (see Annex). This implies that stations are the primary, and fish the secondary sampling units of cluster sampling, as if there is a blending of $\bar{Y}_{\text{geog}}$ and $\bar{Y}_{\text{pop}}$ with respect to sampling units.

The mean over stations (4) is easier to categorise. It gives equal weighting to all stations and therefore has geographical sampling units as for $\bar{Y}_{\text{geog}}$.

The two estimators, (3) and (4), also have different practical implications. Stations where zero fish of a species are caught have no effect on $\bar{y}_{\text{fish}}$. On the other hand, correct counting of n_{stn} to include only those stations where fish were caught is needed for $\bar{y}_{\text{stn}}$. Then, $\bar{y}_{\text{stn}}$ represents the mean over stations for that part of the stock domain found to exist within the survey domain. The two estimators also require different formulae for their sampling variances.

2.4 Sampling variances

The straightforward case is when the indicator measure is determined for all fish in every catch taken on a survey. Then the variance of the mean over stations (4) is simple to estimate if the mean result for each station, $\bar{y}_i$, can satisfactorily be assumed equivalent to an independent, randomly located observation within the domain of the stock. Nicholson et al. (1991) point out that such an assumption is conditional upon the design of the survey. The formula is:

$$\mathrm{var}\left(\bar{y}_{\mathrm{stn}}\right)=\frac{\sum_{i=1}^{n_{\mathrm{stn}}}\left(\bar{y}_i-\bar{y}_{\mathrm{stn}}\right)^2}{n_{\mathrm{stn}}\left(n_{\mathrm{stn}}-1\right)}.$$

Since the mean over fish (3) is a simple average over the fish caught by the survey, its sample variance, at first sight, appears to be $\sum_i\sum_j\left(y_{ij}-\bar{y}_{\mathrm{fish}}\right)^2\Big/n_{\mathrm{fish}}\left(n_{\mathrm{fish}}-1\right)$. However, in practice, this would over-estimate sampling precision because the fish are not independently selected from the stock and the degrees of freedom in the denominator are therefore too high. In other words, it would only be unbiased for the variance if the fish with different attributes were perfectly evenly mixed up within the stock domain which, of course, is highly unlikely. Instead, by treating $\bar{y}_{\mathrm{fish}}$ as a ratio estimator with stations as sampling units, the sample variance can be found with the usual approximation for ratio estimators which, from Thompson (1992, p. 61), is

$$\mathrm{var}_R\left(\bar{y}_{\mathrm{fish}}\right)=\frac{\sum_{i=1}^{n_{\mathrm{stn}}}\left(y_i-\bar{y}_{\mathrm{fish}}n_i\right)^2}{n_{\mathrm{stn}}\left(n_{\mathrm{stn}}-1\right)}$$

where $y_i=\sum_{j=1}^{x_i}y_{ij}$ is the total of the measured attribute over all fish at the i'th station. The variance of a ratio estimate can also be obtained with the jackknife estimator (Cochran 1977, p. 178; Pennington and Vølstad 1994), or by nonparametric bootstrapping (Aanes and Pennington 2003).

The less straightforward case arises when there is only time on the survey to determine the indicator measures for a sample of fish from some or all catches. There is then an additional component of variance. It is automatically included in estimates of $\mathrm{var}\left(\bar{y}_{\mathrm{stn}}\right)$ because within-catch variance appears in the $\bar{y}_i$. However, it is not automatically included in $\mathrm{var}\left(\bar{y}_{\mathrm{fish}}\right)$ because some of the y_i are not measured when catches are sampled. The simplest solution is to work with raised estimates of n_i and y_i at each station and, in the absence of simple theory, ignore the additional sampling variance arising. Nevertheless, published studies of catch sampling variance do exist (Cotter 1998) and may allow development of a more precise approach if needed.

2.5 Properties of the estimators

The properties discussed in this and the preceding section are summarised comparatively in Table 1. The mean over fish (3), though intuitively a good estimator for $\bar{Y}_{\mathrm{pop}}$, may turn out not to be in practice because of the clustering of fish brought about by trawling. It might be biased towards values displayed by the most abundant classes, and it is difficult to use when catches have been sampled. Pennington and co-workers (Pennington and Vølstad 1994; Pennington et al. 2002) point out that within-haul correlation often means that the measurement of thousands of fish on a trawl survey is no more precise than would be obtained with a handful of fish (often less than 10) if it were possible to select them truly randomly from the population. In support of this, Aanes and Pennington (2003) found that the mean over fish was less precise than the mean over stations in 3 out of 4 seasonal market sampling surveys. Another disconcerting aspect of the mean over fish is the irrelevance of the number of stations in the estimation formula. The implication is that the survey can dispense with geographic coverage and need only visit a few stations likely to yield large catches.

Concerning the mean over stations (4), the uniform weighting of stations might be regarded as an advantage, given the problems of knowing how much information is provided by each trawl catch and given that stations can reasonably be thought of as independent observations in a geographic domain. On the other hand, some would feel uneasy about taking the same amount of information from a station yielding only one fish as from another yielding many. This can lead to bias towards values displayed by the least abundant classes. The contrasting characteristics of the mean over fish and the mean over stations suggest that an estimator combining their qualities would be beneficial.

2.6 A compromise: a mixed-model estimator

Choosing a blend of the mean over fish and the mean over stations so as to benefit from knowledge of the numbers of stations and the numbers of fish at each can be assisted by considering the average within-haul correlation. A low degree of correlation, i.e. a wide range of attribute values for the fish at each station, suggests that $\bar{y}_{\mathrm{fish}}$ should have most weighting because more fish observed implies more information about the stock. A high degree, i.e. small attribute values predominate at some stations, large values at others, suggests that $\bar{y}_{\mathrm{stn}}$ should have greater weighting because, in contrast, having more stations implies more information about the stock. This interpretation is consistent with sampling theory which indicates that cluster samples are most efficient when there is much variation within clusters, and little between (Cochran 1977; Sukhatme and Sukhatme 1984; Thompson 1992). It can also be seen from the definition of the within-haul correlation, here adapted from Cochran (1977, p. 209) and referring to the i'th station:

$$\rho_i=\frac{E\left(y_{ij}-\bar{Y}_{\mathrm{pop}}\right)\left(y_{ik}-\bar{Y}_{\mathrm{pop}}\right)}{E\left(y_{ij}-\bar{Y}_{\mathrm{pop}}\right)^2}.$$

The attribute values, y_{ij} and y_{ik}, $j\neq k$, for individual fish at a station will vary around the observed station mean, $\bar{y}_i$, which is not in the formula. At one extreme, $\bar{y}_i=\bar{Y}_{\mathrm{pop}}$, so y_{ij} and y_{ik} also vary randomly around $\bar{Y}_{\mathrm{pop}}$, and $\rho_i=0$. Then, no information is gained from the station means and all information is derived from the fish. More usually, $\bar{y}_i\neq\bar{Y}_{\mathrm{pop}}$. Then, the deviation of $\bar{y}_i$ from $\bar{Y}_{\mathrm{pop}}$ is included in all values of y_{ij} and y_{ik} causing ρ_i to be positive. At the other extreme (which is strange to imagine), every fish caught at the i'th station has exactly the same attribute value. Then all $y_{ij}=y_{ik}=\bar{y}_i\neq\bar{Y}_{\mathrm{pop}}$,

Table 1. Summarised comparison of the properties of two intuitive estimators for the mean value of an indicator, and a third based on fitting a mixed linear model. See text for estimators.

Property	Mean over fish	Mean over stations	Mixed model
Sampling units	Fish, OR (with ratio est.) stations	Stations	Mix of fish AND stations
Sampling variance based on:	Variance of a ratio estimator, jackknife, or bootstrap	Variance of station means, depending on survey design	Mix of estimated variances between- and within-stations
Weighting of stations:	Highest for stations with large catches	Equal regardless of catch size	Unbalanced catch sizes are allowed for.
Sensitive to stations with zero catches	No	Yes	Yes
Effect on time series of high intra-haul correlation	Increases variance, and biases towards attributes of most abundant fish class	Increases variance	Little or none: effect is removed
Effect of unbiased catch sampling	Catch samples must be raised to estimate true catch before estimation	No estimation problem but adds extra component of sampling variance	Catch samples must be raised to estimate true catch before estimation
Effect of a +ve correlation between the indicator and abundance in a catch	Abundant classes of the species have most influence on indicator	Scarce classes have relatively more influence if abundant classes do not occur at all stations	Increases between- and within-station variances if abundant classes do not occur at all stations

and $\rho_i = 1$. In words, no information is gained by observing the attribute on more than one fish at each station, and all variability is geographic.

The proposed compromise mean involves, firstly, fitting a mixed model (Aanes and Pennington 2003) to all the measurements made on each annual survey:

$$y_{ij} = \mu + y'_i + e_{ij} \tag{5}$$

where μ is the fixed-effect overall mean for the year, $y'_i = \bar{y}_i - \mu$ is the random station effect with $y'_i \sim N\left(0, \sigma^2_{\text{stn}}\right)$, and $e_{ij} \sim N\left(0, \sigma^2_{\text{e}}\right)$ is the random deviation of the j'th individual fish at the i'th station. The R lme() function can be used to fit such models without concern for different numbers of observations in each group (Pinheiro and Bates 2000, p. 26) meaning, in the present context, different numbers of fish caught at each station.

Now, assuming – as if in a perfect world – that model (5) is well identified and sampling is unbiased, two of the terms in (5) can be related to sampling terms in (4). Firstly,

$$\mu = E\left(\bar{y}_{\text{stn}}\right)$$

because, if this were not true, y'_i in (5) would not have a zero mean as defined. Secondly, the station effect

$$y'_i = \bar{y}_i - E\left(\bar{y}_{\text{stn}}\right)$$

if all fish are measured at each station. From theory (Searle et al. 1992, p. 12; Pinheiro and Bates 2000, p. 227), the average within-group correlation for independently and identically distributed within-catch errors is:

$$\rho = \frac{\sigma^2_{\text{stn}}}{\sigma^2_{\text{stn}} + \sigma^2_{\text{e}}}. \tag{6}$$

It can be seen for the two possible extreme conditions: (1) $\sigma^2_{\text{stn}} = 0$ (with $\sigma^2_{\text{e}} > 0$) corresponds with $\rho = 0$, implying that $\bar{y}_{\text{fish}}$ is the appropriate estimator; and (2) $\sigma^2_{\text{e}} = 0$ (with $\sigma^2_{\text{stn}} > 0$) corresponds with $\rho = 1$, implying that $\bar{y}_{\text{stn}}$ is then appropriate. Accordingly, the proposed mixed model, compromise estimator is

$$\bar{y}_{\text{mix}} = \hat{\rho}\bar{y}_{\text{stn}} + (1 - \hat{\rho})\,\bar{y}_{\text{fish}} \tag{7}$$

where $\hat{\rho}$ is calculated by substituting the mixed model variance estimates, $\hat{\sigma}^2_{\text{stn}}$ and $\hat{\sigma}^2_{\text{e}}$ into (6). The variance of (7) appears not to be simple to formulate exactly. Bootstrapping an estimate may be feasible. For the present, the following intuitive estimator is offered as a simple guide to precision. A covariance term is omitted thus implying that $\bar{y}_{\text{stn}}$ and $\bar{y}_{\text{fish}}$ vary independently around $\bar{Y}_{\text{pop}}$ – as might have some justification given that the numbers caught at each station are random. A term for $\text{var}\,(\hat{\rho})$ is also omitted, implying that $\hat{\rho}$ is accurately estimated– as might be justified by large numbers of stations and fish used to fit (5). The suggested estimator is

$$\text{var}\left(\bar{y}_{\text{mix}}\right) \approx \hat{\rho}^2 \text{var}\left(\bar{y}_{\text{stn}}\right) + (1 - \hat{\rho})^2\, \text{var}\left(\bar{y}_{\text{fish}}\right). \tag{8}$$

For $\text{var}\,(\bar{y}_{\text{stn}})$, substitute estimated parameters, denoted with ˆ, into (5):

$$y_{ij} = \hat{\mu} + \hat{y}'_i + \hat{e}_{ij}$$

then substitute for y_{ij} in (4). All terms are assumed independent (as was necessary for fitting the mixed model). On simplifying the summations, this gives

$$\hat{\bar{y}}_{\rm stn} = \hat{\mu} + n_{\rm stn}^{-1} \sum_i \hat{y}'_i + n_{\rm stn}^{-1} \sum_i n_i^{-1} \sum_j \hat{e}_{ij}$$

for which the variance estimator, after squaring coefficients and expanding and simplifying summations, is

$$\mathrm{var}\left(\hat{\bar{y}}_{\rm stn}\right) = \mathrm{var}\left(\hat{\mu}\right) + n_{\rm stn}^{-1}\hat{\sigma}_{\rm stn}^2 + n_{\rm stn}^{-2}\hat{\sigma}_{\rm e}^2 \sum_i n_i^{-1}. \qquad (9)$$

From this, we can note in passing that the numbers of fish caught at each station and the variance of their measured attribute around the station mean contribute to $\mathrm{var}\left(\hat{\bar{y}}_{\rm stn}\right)$ even though the number of fish caught at each station does not affect $\bar{y}_{\rm stn}$ estimated with formula (4).

Working similarly for $\mathrm{var}\left(\bar{y}_{\rm fish}\right)$, substituting estimated parameters for (5) into (3) gives

$$\hat{\bar{y}}_{\rm fish} = \hat{\mu} + n_{\rm fish}^{-1} \sum_i n_i \hat{y}'_i + n_{\rm fish}^{-1} \sum_i \sum_j \hat{e}_{ij}$$

from which

$$\mathrm{var}\left(\hat{\bar{y}}_{\rm fish}\right) = \mathrm{var}\left(\hat{\mu}\right) + n_{\rm fish}^{-2}\hat{\sigma}_{\rm stn}^2 \sum_i n_i^2 + n_{\rm fish}^{-1}\hat{\sigma}_{\rm e}^2. \qquad (10)$$

Note in passing that the variance among stations contributes to this expression even though the number of stations does not affect $\bar{y}_{\rm fish}$ estimated with formula (3). Expanding (8) with (9) and (10) gives

$$\mathrm{var}\left(\bar{y}_{\rm mix}\right) \approx \mathrm{var}\left(\hat{\mu}\right) + \hat{\rho}^2 \left\{ n_{\rm stn}^{-1}\hat{\sigma}_{\rm stn}^2 + n_{\rm stn}^{-2}\hat{\sigma}_{\rm e}^2 \sum_i n_i^{-1} \right\} + (1-\hat{\rho})^2 \left\{ n_{\rm fish}^{-2}\hat{\sigma}_{\rm stn}^2 \sum_i n_i^2 + n_{\rm fish}^{-1}\hat{\sigma}_{\rm e}^2 \right\}. \qquad (11)$$

Properties of the mixed model estimation method are proposed in Table 1 for comparison with those of the mean over fish and over stations.

3 Example application

As an aid to assessing the reasoning about estimators presented here, all three were used to estimate mean body length of cod (*Gadus morhua*) from haul-by-haul length distributions taken from the 3rd quarter English groundfish survey (EGFS) of the North Sea. Only data from 1992 to 2007 were analysed because, previously, a different trawl was used. Little or no catch sampling was carried out for cod on the survey because the species was not over-abundant for complete processing. The length data were stored as a data frame, here called COD.L, with columns for *Year*, *Station*, and *Length*, and one row per fish. The lme() function... (Pinheiro and Bates 2000) in the R language was used to fit the model, one year at a time, with the command line:

```
> model92 <- lme(Length ~ 1, data = COD.L, random =
        ~ 1|Station, subset = Year == 1992)
```

The first "~1" indicates that the model has one fixed effect, the intercept, equivalent to $\bar{y}_{\rm stn}$. The "random = ~1" argument indicates that *Station*, equivalent to y'_i, varies randomly around the intercept.

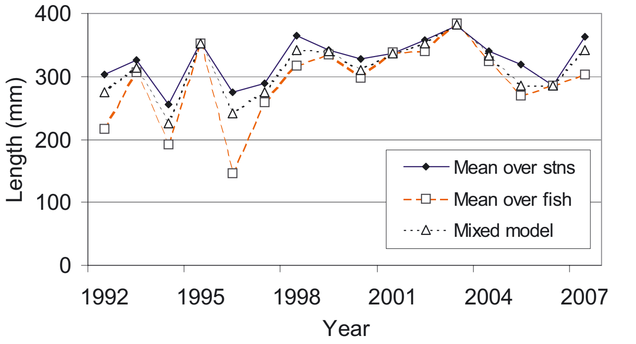

Fig. 1. Cod from the North Sea: annual mean length as determined by 3 different estimators applied to length data obtained by the English groundfish survey.

Figure 1 shows $\bar{y}_{\rm fish}$, $\bar{y}_{\rm stn}$, and $\bar{y}_{\rm mix}$. $\bar{y}_{\rm stn}$ was less variable from year to year than $\bar{y}_{\rm fish}$. The latter appears to have been affected by recruitments of large numbers of juvenile fish. To explore this, data on the abundance of cod less than 160 mm in length are provided (Table 2). These juveniles were observed to be much less abundant from 1997 onwards, the same period as $\bar{y}_{\rm fish}$ rose to levels that were comparable with those of the other two estimators. Comparable events occurred in 1993 and 1995. The mostly lower level of $\bar{y}_{\rm fish}$ compared to $\bar{y}_{\rm stn}$ is therefore consistent with the view that $\bar{y}_{\rm fish}$ emphasises the abundant age classes, and that $\bar{y}_{\rm fish}$ is therefore likely to indicate more variability than $\bar{y}_{\rm stn}$, e.g. the upward trend between 1996 and 1999 occurred because of the large number of juveniles present in 1996.

Referring again to Figure 1, $\bar{y}_{\rm mix}$ always fell between the other two means as expected from (7). Given that it is based on the within-haul correlation, the positions should reflect the average degree of independence of measurements within each haul. Table 3 shows output from fitting the mixed model that was used for estimating the standard error of $\bar{y}_{\rm mix}$ with (11). The declining abundance of cod is evident from the declining numbers of cod measured. The standard errors for $\bar{y}_{\rm mix}$ are mostly smaller than those for $\bar{y}_{\rm fish}$ and $\bar{y}_{\rm stn}$, suggesting that the information available from the distribution of fish by station, and from the total numbers caught, have been combined successfully to benefit precision.

4 Discussion

The "mean" value of an indicator can be many things and different sorts of mean can show different values and trends over time for purely statistical reasons related to the number and attributes of fish caught at each fishing station. The central issue explored here concerns the relative weighting of the variability among individuals and among locations. The mixed mean appears to offer a general-purpose and efficient estimator that finds a justifiable balance between these two sources of variability through application of the intra-haul correlation. It is therefore finding more information from the survey than either the mean over fish or the mean over stations.

Table 2. English groundfish survey of the North Sea: abundance of juvenile cod observed per hour of trawling in 10-mm length groups up to 150–160 mm.

Year	Length (mm)											
	40	50	60	70	80	90	100	110	120	130	140	150
1992			18	108	239	293	207	165	57	8	3	1
1993		2	7	14	33	65	43	51	12	6		1
1994	4	42	141	219	291	287	215	116	44	5		2
1995		1	5	2	8		2	1				
1996	1	31	273	1040	1085	228	105	52	15	1	1	2
1997	1	1	1								11	21
1998		1		2	16	15	17	21	22	22	39	21
1999		1	6	5	7	14	10	13	1	2	3	1
2000		1	2	4	8	4	3	2	1			
2001		1	2	6	1	4	1					
2002		1			3	4	3	1				1
2003			1	1	1	2		1	1	1	2	
2004				2		2	2	5	1			
2005	1	3	11	29	29	23	13	4	4	3	2	2
2006			3	1	10	6	5	3	3	4	4	2
2007		1	1		10	12	35	50	42	31	38	27

Table 3. English groundfish survey of the North Sea: selected output from fitting a mixed model to body lengths (mm) of cod as needed for estimating standard errors of $\bar{y}_{\text{mix}}$. Notation is given in the text.

Year	N cod measured	n_{stn}	$\bar{y}_{\text{mix}}$	$\hat{\sigma}_{\text{stn}}$	$\hat{\sigma}_e$	st.err $(\hat{\mu}_{\text{stn}})$	st.err $\left(\hat{\bar{y}}_{\text{stn}}\right)$	st.err $\left(\hat{\bar{y}}_{\text{fish}}\right)$	$\hat{\rho}$	st.err $(\bar{y}_{\text{mix}})$
1992	2636	63	275	124	85	16	23	42	0.68	20.6
1993	1060	57	313	111	134	16	23	29	0.41	19.7
1994	3133	65	226	134	106	17	24	31	0.62	19.2
1995	1526	68	352	108	105	14	20	25	0.51	15.9
1996	3733	66	242	152	85	19	27	90	0.76	29.5
1997	2625	73	274	84	73	10	14	18	0.57	11.4
1998	1197	55	342	84	76	12	17	26	0.55	15.0
1999	448	53	340	125	119	20	29	39	0.52	23.9
2000	754	62	311	103	98	15	21	29	0.52	17.7
2001	466	53	337	86	91	14	20	23	0.47	15.1
2002	597	55	353	125	85	18	26	38	0.68	21.3
2003	244	44	382	111	134	20	29	31	0.41	21.9
2004	338	49	333	109	119	18	26	35	0.45	22.6
2005	372	45	285	116	131	20	29	35	0.44	23.2
2006	811	57	286	66	104	11	16	26	0.28	19.5
2007	904	55	342	146	101	21	30	45	0.67	25.0

While this paper considered types of mean as the primary summarising statistic, quantiles may be preferred for summarising the results of some indicator studies, perhaps to minimise the influence of outlying values. Quantiles, like the mean, can be estimated over fish, or over the fish at each station then averaged over stations. The averaging over stations could also be done using quantiles if there are enough stations. The statistical issues appear to be closely comparable to those discussed here in connection with the mean over fish and over stations, and there is a similar need for clear reporting of the method used to estimate quantiles.

Trawl surveys are often stratified geographically. The stratified random sampling estimators from sampling theory are applicable to finite populations and so use the numbers of sampling units for estimates of the means and variances of the individual strata and the overall population. In the context of trawl surveys, however, the issues discussed in this paper imply that there can be differing interpretations of the meaning of "sampling unit" and therefore of the relative population sizes in different strata. Estimation of the stratum means and variances can use whichever of the formulae presented here are preferred. However, in estimating results for the overall population, the weighting factors for each stratum should be logically consistent. So, a preference for the mean over fish for the stratum means implies that strata should be weighted by the numbers of fish caught in each to estimate the population mean, assuming that they are the best available estimates of population sizes. A preference for the mean over stations, on the other hand, implies that strata should be weighted by their geographic areas. Cochran (1977) discusses the effects on estimation of errors in stratum sizes. The mixed model estimator might be developed for use in stratified surveys by including an additional random term for stratum variability.

The domain occupied by a species is an important yet frequently unstated qualifier for an estimated mean. The species domain within a survey domain can be estimated naturally by finding all fixed station locations where one or more individuals have ever been found, or, in the case of a stratified random survey, all strata. This has been done for commercial species surveyed by the international bottom trawl survey of the North Sea, for example (Anonymous 2004a). Since some species expand and contract the area they occupy in relation to abundance, or may move in response to climatic or other factors, updating the boundaries of the stock domain regularly could provide another informative indicator of the state of the stock. The task would present its own sampling problems because lack of fish in a single catch does not necessarily mean that fish were not present in the vicinity of the trawl. Spatial indicators are likely to be valuable for this kind of study (Woillez et al. 2007, and elsewhere in this issue).

Acknowledgements. I am grateful to Jacques Rivoirard, Verena Trenkel, Pierre Petitgas, and the referees for raising several issues that needed more care. They are not responsible for remaining errors. The study was funded by the EC FISBOAT project and by the Department of Environment, Food and Rural Affairs of the UK government.

Appendix

Demonstration that the mean over fish is an approximation to the mean estimated by single-stage cluster sampling

The following is based on formulae given by Thompson (1992). In cluster sampling, the population mean for the secondary sampling units composing each cluster is $\mu = Y/N$ where $Y = \sum_{i=1}^{N_{\rm stn}} \sum_{j=1}^{N_i} y_{ij}$ and $N = \sum_{i=1}^{N_{\rm stn}} N_i$.

Here, stations are being taken as the primary units counted by $i = 1, \ldots, N_{\rm stn}$; fish are the secondary units counted by $j = 1, \ldots, N_i$ for the i'th station; and y_{ij} is the value of the measured attribute on the j'th fish at the i'th station. $N_{\rm stn}$ is the number of potential fishing stations within the survey region. Estimators of Y and N, based on a survey, are:

$$\hat{Y} = \frac{N_{\rm stn}}{n_{\rm stn}} \sum_{i=1}^{n_{\rm stn}} \sum_{j=1}^{n_i} y_{ij}$$

and $\hat{N} = \frac{N_{\rm stn}}{n_{\rm stn}} \sum_{i=1}^{n_{\rm stn}} n_i$.

Then the unknown $N_{\rm stn}$ cancels in the estimator for the mean of the attribute:

$$\hat{\mu} = \frac{\hat{Y}}{\hat{N}} = \frac{\sum_{i=1}^{n_{\rm stn}} \sum_{j=1}^{n_i} y_{ij}}{\sum_{i=1}^{n_{\rm stn}} n_i} = \bar{y}_{\rm fish}.$$

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